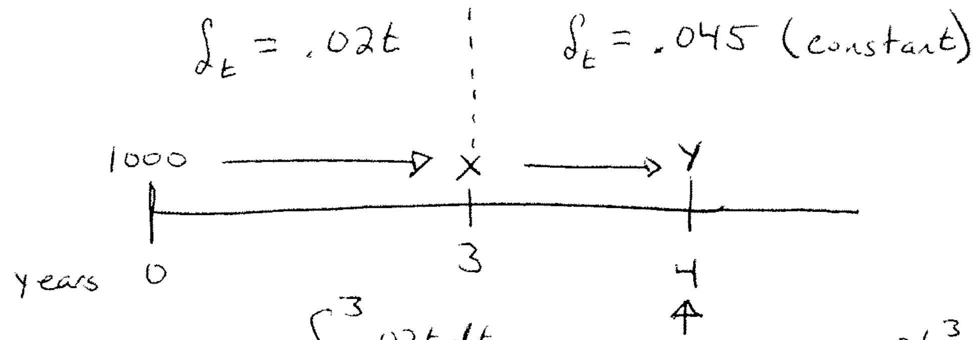
37/ (May 2000 Course 2)




$$X = 1000\, e^{\int_0^3 .02t\,dt} = 1000\, e^{.01t^2\big|_0^3} = 1000 e^{.09}$$

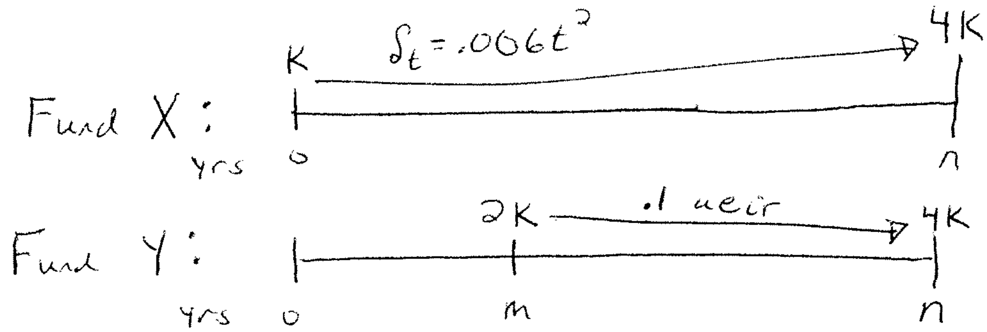
$$Y = X \cdot e^{.045} = 1000 e^{.09} e^{.045} = 1000 e^{.135}$$

$$i = \text{qeir} \implies 1000(1+i)^{16} = Y = 1000\, e^{.135}$$

$$\implies i = e^{\frac{.135}{16}} - 1$$

$$\therefore \boxed{i^{(4)} = 4i \doteq 0.03389}$$

53/ (Nov. 2000 Course 2)




$$\text{Fund X:} \quad 4K = K\, e^{\int_0^n .006t^2\,dt} = K\, e^{.002t^3\big|_0^n} = K\, e^{.002n^3}$$

$$\therefore e^{.002n^3} = 4$$

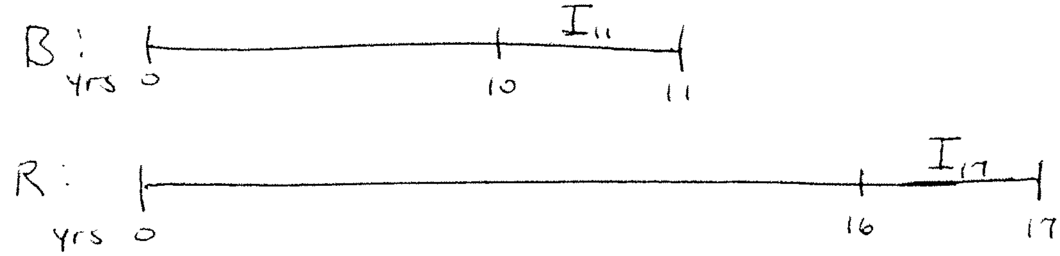
$$\text{Fund Y:} \quad 2K(1.1)^{n-m} = 4K \implies (1.1)^{n-m} = 2$$

$$\implies n - m = \frac{\ln(2)}{\ln(1.1)} \implies m = n - \frac{\ln(2)}{\ln(1.1)}$$

$$n = \left(\frac{\ln(4)}{.002}\right)^{1/3} \implies \boxed{m \doteq 1.57743}$$

12/May 2001 Course 2)

$a(t) = (1-d)^t$ t-yrs




$$B: I_{11} = d \cdot A(11) = d \cdot 100(1-d)^{-11} = X$$

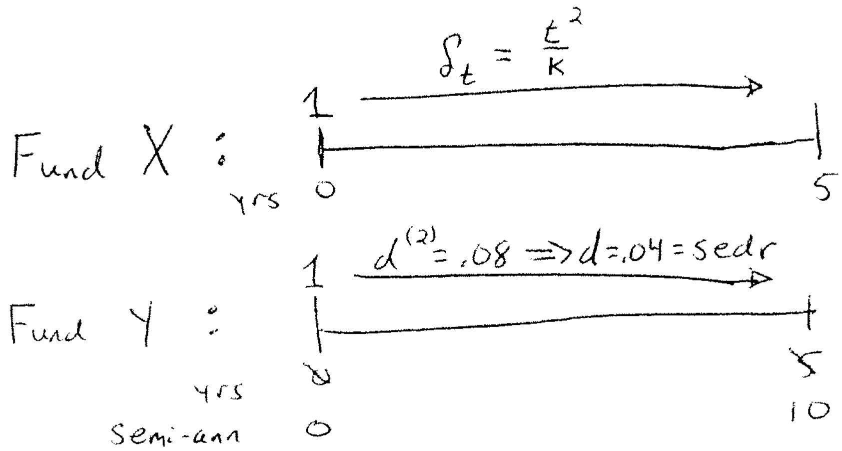
$$R: I_{17} = d \cdot A(17) = d \cdot 50(1-d)^{-17} = X$$

$$\therefore \quad 100(1-d)^{-11} = 50(1-d)^{-17}$$

$$\Longrightarrow (1-d)^6 = \tfrac{1}{2} \Longrightarrow d = 1 - \left(\tfrac{1}{2}\right)^{1/6}$$

$$\therefore \boxed{X = d(100)(1-d)^{-11} \doteq 38.87928}$$

45/May 2001 Course 2)




$$\therefore \quad e^{\int_0^5 \frac{t^2}{K}\,dt} = (1-.04)^{-10} = (.96)^{-10}$$

$$\int_0^5 \frac{t^2}{K}\,dt = \frac{t^3}{3K}\Big|_0^5 = \frac{125}{3K}$$

$$\therefore \quad e^{\frac{125}{3K}} = (.96)^{-10} \Longrightarrow \frac{125}{3K} = -10\ln(.96)$$

$$\Longrightarrow \boxed{K = \frac{125}{-30\ln(.96)} \doteq 102.06916}$$

49 / May 2001 Course 2) Since we're measuring forces of interest, we need time measured in years for both accounts.

$$T: \; i^{(2)} = 10\% \implies \text{seir} = .05$$

$$i = \text{aeir} \implies 1+i = (1.05)^2 = 1.1025$$

$$\therefore \; a(t) = (1.1025)^t \quad t\text{ - years}$$

$$a'(t) = (1.1025)^t \cdot \ln(1.1025)$$

$$\implies \delta_t = \frac{a'(t)}{a(t)} = \ln(1.1025) \quad (\text{constant})$$

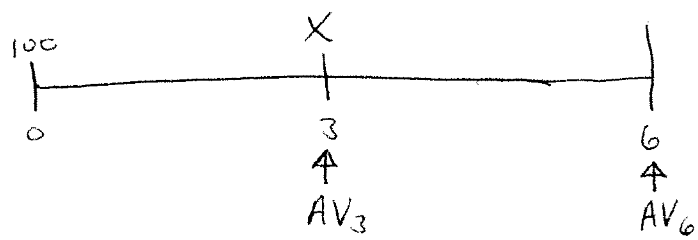
$$F: \; a(t) = 1 + it \qquad i\text{ - simple}$$

$$a'(t) = i \implies \delta_t = \frac{i}{1+it}$$

$$\therefore \text{ at time } 5, \; \frac{i}{1+5i} = \ln(1.1025) \implies i = \frac{\ln(1.1025)}{1 - 5\ln(1.1025)}$$

$$\boxed{Z = 1000(1+5i) \doteq 1952.74987}$$

1 / Nov. 2001 Course 2)




$$\delta_t = \frac{t^2}{100}$$

$$AV_3 = 100\, e^{\int_0^3 \frac{t^2}{100}dt} + X = 100\, e^{\frac{t^3}{300}\Big|_0^3} + X = 100\, e^{\frac{27}{300}} + X = 100\, e^{.09} + X$$

$$AV_6 = 100\, e^{\int_0^6 \frac{t^2}{100}dt} + X e^{\int_3^6 \frac{t^2}{100}dt} = 100\, e^{\frac{t^3}{300}\Big|_0^6} + X e^{\frac{t^3}{300}\Big|_3^6} = 100\, e^{.72} + X e^{.63}$$

$$I_{[3,6]} = AV_6 - AV_3 = X$$

$$\therefore \; (100\, e^{.72} + X e^{.63}) - (100\, e^{.09} + X) = X$$

$$\implies \boxed{X = \frac{100\, e^{.72} - 100\, e^{.09}}{2 - e^{.63}} \doteq 784.59308}$$

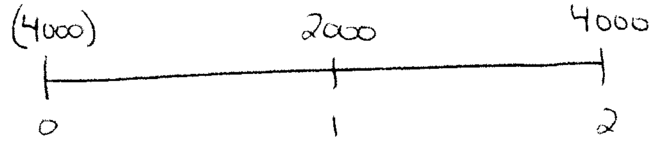
24/ Nov. 2001 Course 2)  $100 + 200v^n + 300v^{2n} = 600v^{10}$

$v^n = 0.75941 \qquad v^{2n} = (v^n)^2$

$\therefore\ 600v^{10} \doteq 424.89306$

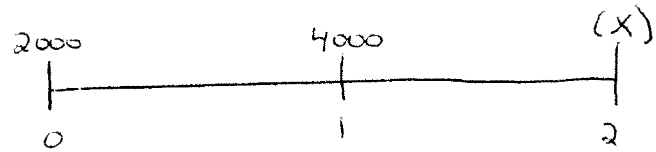
$\Longrightarrow (1+i)^{10} \doteq \dfrac{600}{424.89306} \Longrightarrow \boxed{i \doteq .03511}$

47/ Nov. 2001 Course 2)  P :




$NPV = -4000 + 2000v + 4000v^2$

Q :




$NPV = 2000 + 4000v - Xv^2$

Using $i = 10\%$, $v = \frac{1}{1.1}$ and $NPV(P) = NPV(Q)$

$-4000 + 2000v + 4000v^2 = 2000 + 4000v - Xv^2$

$\Longrightarrow X = (6000 + 2000v - 4000v^2)(1.1)^2 \quad \left[\frac{1}{v^2} = (1.1)^2\right]$

$\therefore \boxed{X = 5460}$

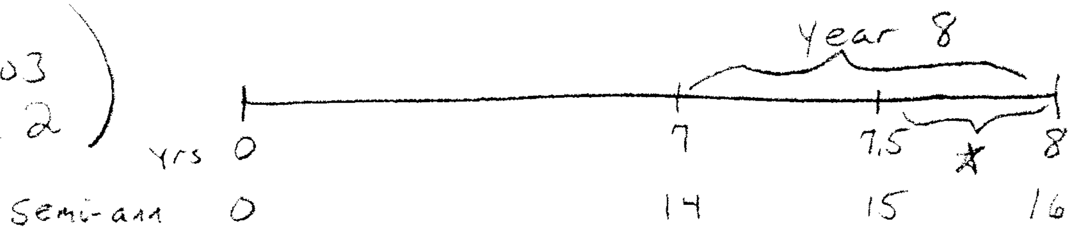
1/ May 2003 Course 2)  Note: The $i$ in this problem is what we call $i^{(2)}$.

$B: 100\left(1+\frac{i}{2}\right)^{14.5} = 200 \Longrightarrow i = .097929$

$P: 100e^{7.25\delta} = 200 \Longrightarrow \delta = .095607$

$\therefore \boxed{i - \delta \doteq .002322}$

12/ May 2003 Course 2)




$$E:\ I_{[15,16]} = AV_{16} - AV_{15} = X(1+\tfrac{i}{2})^{16} - X(1+\tfrac{i}{2})^{15}$$

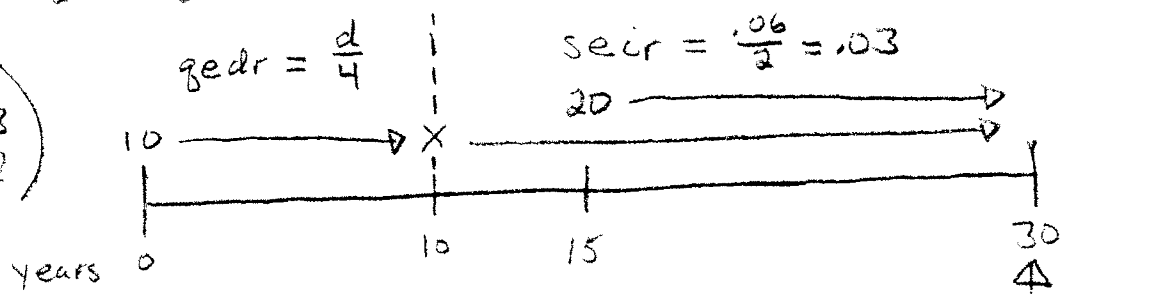
$$\underline{\underline{\text{factor}}}\ X(1+\tfrac{i}{2})^{15}\left[(1+\tfrac{i}{2})-1\right] = X(1+\tfrac{i}{2})^{15}\cdot\tfrac{i}{2}$$

$$M:\ I_{[7.5,8]} = 2X(1+8i) - 2X(1+7.5i)$$

$$= 2X\left[(1+8i)-(1+7.5i)\right] = 2X\cdot(0.5i) = Xi$$

$$\therefore\ X(1+\tfrac{i}{2})^{15}\cdot\tfrac{i}{2} = Xi \Longrightarrow (1+\tfrac{i}{2})^{15} = 2 \Longrightarrow \boxed{i \doteq .094588}$$

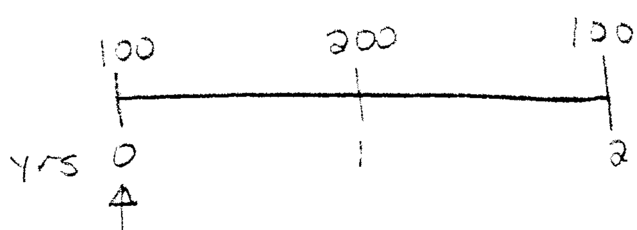
50/ May 2003 Course 2)




$$100 = \underbrace{10(1-\tfrac{d}{4})^{-40}}_{X}\cdot(1.03)^{40} + 20(1.03)^{30}$$

$$\therefore\ \boxed{d = .045318}$$

7/ May 2005 Exam FM)




$$PV = 364.46 = 100 + 200v + 100v^2 \quad v = \text{adf}$$

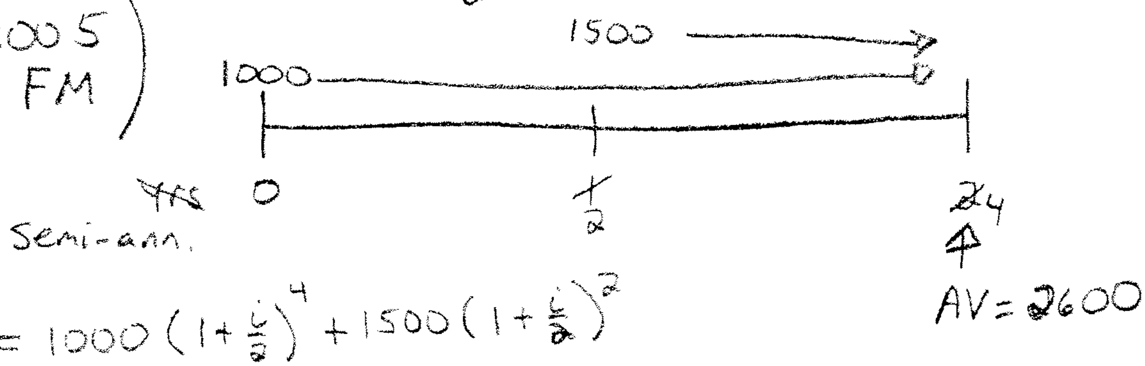
$$\therefore\ 100v^2 + 200v - 264.46 = 0 \quad (\text{quadratic in } v)$$

$$\therefore\ v = \frac{-200 \pm \sqrt{200^2 - 4(100)(-264.46)}}{200} \doteq .90908$$

$$i = \text{aeir} \Longrightarrow v = \frac{1}{1+i} \Longrightarrow \boxed{i \doteq .10}$$

13/ May 2005 Exam FM

$seir = \frac{i}{2}$




$$2600 = 1000\left(1+\frac{i}{2}\right)^4 + 1500\left(1+\frac{i}{2}\right)^2$$

quadratic in $\left(1+\frac{i}{2}\right)^2$  $a = 1000$, $b = 1500$, $c = -2600$

$$\left(1+\frac{i}{2}\right)^2 = \frac{-1500 \pm \sqrt{1500^2 - 4(1000)(-2600)}}{2000} \implies \boxed{i \doteq .028144}$$

18/ May 2005 Exam FM

$aeir = .08 \implies v = adf = \frac{1}{1.08}$

$P$ = purchase price

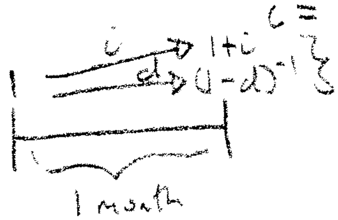
$$\left.\begin{array}{l} PV(\text{Option } 1) = .9P \cdot v^{2/12} = .9Pv^{1/6} \\ PV(\text{Option } 2) = P - .01X \cdot P = P\left(1 - \frac{X}{100}\right) \end{array}\right\} =$$

$$\therefore .9v^{1/6} = 1 - \frac{X}{100} \implies \boxed{.9 = \left(1-\frac{X}{100}\right)(1.08)^{1/6}}$$

19/ May 2005 Exam FM

$i^{(12)} = .189$  Determine $d^{(12)}$

$i = \frac{.189}{12} = .01575 = meir$  $d = \frac{d^{(12)}}{12} = medr$




$\implies 1.01575 = (1-d)^{-1}$

$$\therefore d = 1 - (1.01575)^{-1} \implies \boxed{d^{(12)} = 12d = .18607}$$

7/ Nov. 2005 Exam FM

First determine maximum amount after 6-year period.
E.g. reinvesting each year gives $AV_6 = 10000(1.01)^{24} = 10697.35$

We see maximum amount is obtained by using a 5-year period and a 1-year period. The maximum amount is $AV_6 = 10000\left(1+\frac{.0565}{4}\right)^{20}(1.01)^4$

$i$ = maximum aeir $\implies 10000(1+i)^6 = 10\,000\left(1+\frac{.0565}{4}\right)^{20}(1.01)^4$

$$\implies \boxed{i \doteq .05484}$$

25/Nov. 2005 Exam FM) X will be paid in 17, 15, and 12 years to the children ages 1, 3, and 6, respectively.

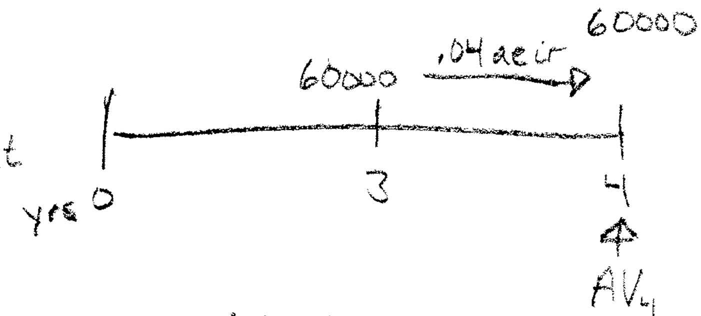
$$PV(\text{payments of } X) = Xv^{17} + Xv^{15} + Xv^{12} = X(v^{17}+v^{15}+v^{12})$$

Y will be paid in 20, 18, and 15 years to the children ages 1, 3, and 6, respectively.

$$PV(\text{payments of } Y) = Yv^{20} + Yv^{18} + Yv^{15} = Y(v^{20}+v^{18}+v^{15})$$

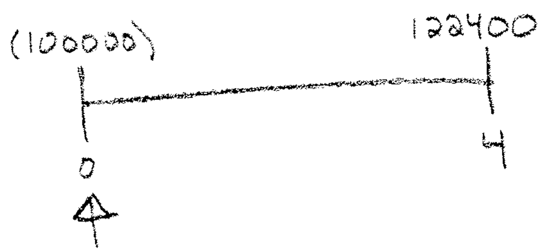
$$\boxed{Z = \text{total } PV = X(v^{17}+v^{15}+v^{12}) + Y(v^{20}+v^{18}+v^{15})}$$

32/Exam FM Sample Questions) Return on Investment




$$AV_4 = 60000(1.04) + 60000 = 122{,}400$$

i.




$$NPV_{.05} = -100000 + 122400\, v^4_{.05}$$

$$= -100000 + 122400 \cdot \frac{1}{(1.05)^4} \doteq \boxed{698.78}$$

61/Exam FM Sample Questions) $f(t) = 3 + \frac{t^3}{150} \Rightarrow f'(t) = \frac{3t^2}{150} = \frac{t^2}{50}$

Since $\frac{t^2}{100} = \frac{1}{2} \cdot \frac{t^2}{50}$, $\delta_t = \frac{1}{2} \cdot \frac{f'(t)}{f(t)}$

$$\Rightarrow a(t) = \left[\frac{f(t)}{f(0)}\right]^{1/2} = \left(\frac{3 + \frac{t^3}{150}}{3}\right)^{1/2} = \sqrt{1 + t^3/450}$$

$500 \longrightarrow 500 \cdot a(n) = 2000 \Rightarrow a(n) = 4$

0 ——— n

$\therefore \sqrt{1 + n^3/450} = 4 \Rightarrow n = 18.9$

77/Exam FM Sample Questions) Let $n$ = the ~~desired~~ required # of months

Then $\frac{n}{6}$ = the required # of semiannual periods

$$L: AV = 1000\left(1 + \frac{.06}{2}\right)^{n/6} = 1000(1.03)^{n/6}$$

$$D: AV = 1000\left(1 + \frac{.03}{12}\right)^{n} = 1000(1.0025)^{n}$$

$$\therefore \cancel{1000}(1.03)^{n/6} = 2 \cdot \cancel{1000}(1.0025)^{n}$$

$$\Rightarrow \ln(1.03)^{n/6} = \ln(2) + \ln(1.0025)^{n}$$

$$\Rightarrow \frac{n}{6} \cdot \ln(1.03) - n\ln(1.0025) = \ln(2)$$

$$\Rightarrow n = \frac{\ln(2)}{\frac{1}{6}\ln(1.03) - \ln(1.0025)} \doteq 285.29 \text{ months}$$

$$\doteq 47.55 \text{ semiannual periods}$$

Since interest is credited in Lucas's account only at the end of each semiannual period, it takes 48 semiannual periods (or $48(6) = 288$ months) for his account value to be at least double Danielle's account value. Answer: 288

79/ Exam FM Sample Questions) $B: AV_4^B = 10\left(1+\frac{K}{25}\right)^4$

$J: f(t) = K + 0.25t \Rightarrow f'(t) = 0.25$. Since $1 = 4\cdot(0.25)$,

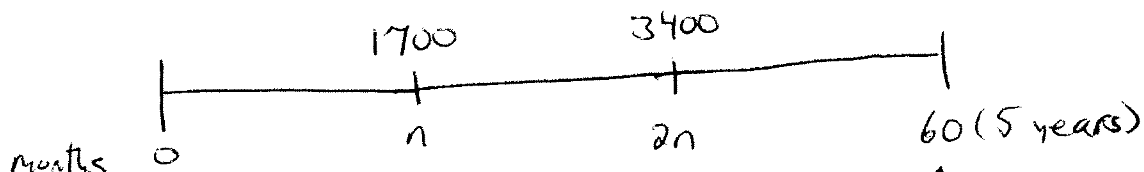
$$\delta_t = 4\cdot\frac{f'(t)}{f(t)} \Rightarrow a(t) = \left[\frac{f(t)}{f(0)}\right]^4 = \left(\frac{K+0.25t}{K}\right)^4 = \left(1+\frac{.25t}{K}\right)^4$$

$$\therefore AV_4^J = 10\left(1+\frac{1}{K}\right)^4$$

$$AV_4^B = AV_4^J \Rightarrow \cancel{1}+\frac{K}{25} = \cancel{1}+\frac{1}{K} \Rightarrow K = 5$$

$$\therefore X = 10\left(1+\frac{1}{5}\right)^4 = 20.736$$

94/ Exam FM Sample Questions) $i = \frac{.072}{12} = .006 = \text{meir}$




$$AV = 1700(1.006)^{60-n} + 3400(1.006)^{60-2n}$$

$$\therefore 1700(1.006)^{60-n} + 3400(1.006)^{60-2n} \geq 6500$$

I would use guess & check at this point. When $n=11$, $AV > 6500$, but when $n=12$, $AV < 6500$. $\therefore n=11$ is the answer.

Note: By writing $(1.006)^{60-n} = (1.006)^{60}\cdot v^n$ and $(1.006)^{60-2n} = (1.006)^{60}\cdot v^{2n}$, we get a quadratic in $v^n$. Then we could solve the quadratic. I think guess and check is easier in this case.

95/Exam FM Sample Questions)

$$\left(1-\frac{d}{2}\right)^{-2} = 1+i$$

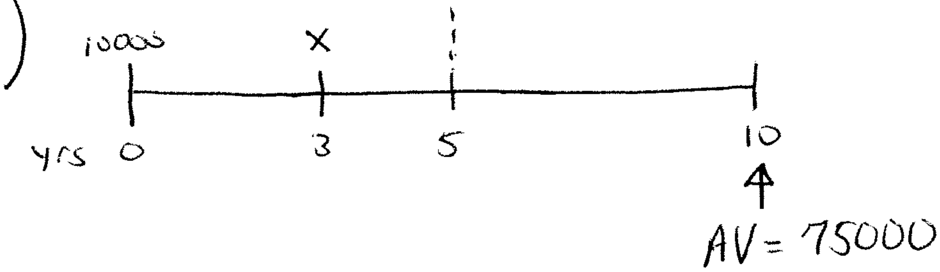
$$S = 1000\left(1-\frac{d}{2}\right)^{-4} \qquad T = 1000\left(1-\frac{d}{4}\right)^{-4}$$

$$\therefore \quad \frac{S}{T} = \frac{\left(1-\frac{d}{2}\right)^{-4}}{\left(1-\frac{d}{4}\right)^{-4}} = \left(\frac{1-\frac{d}{4}}{1-\frac{d}{2}}\right)^{4} = \left(\frac{39}{38}\right)^{4}$$

$$\Longrightarrow 38\left(1-\frac{d}{4}\right) = 39\left(1-\frac{d}{2}\right) \Rightarrow d = .1 \Rightarrow i \doteq .108$$

105/Exam FM Sample Questions)

$i = .06$ aeir | $\delta_t = \frac{1}{t+1} \Rightarrow a(t) = t+1$




$$\therefore \quad 75000 = 10000(1.06)^5 \cdot \frac{a(10)}{a(5)} + X(1.06)^2 \cdot \frac{a(10)}{a(5)}$$

$$\frac{a(10)}{a(5)} = \frac{11}{6}$$

$$\Longrightarrow X \doteq 24498.79$$